Desenhando com transformações lineares

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Precisa-se de um palpite lotérico

Considere os pontos:

$P_4 = (x, y)$

$P_2 = \left(\frac{1}{2}, 1\right)$

$P_1 = (0, 0)$

$P_3 = (1, 0)$

Qual a lógica da coisa?

$$T_1(x,y) = \left(\frac{x + x_1}{2}, \frac{y + y_1}{2}\right), \quad T_2(x,y) = \left(\frac{x + x_2}{2}, \frac{y + y_2}{2}\right), \quad T_3(x,y) = \left(\frac{x + x_3}{2}, \frac{y + y_3}{2}\right)$$

$$T_1(x,y) = \left(\frac{x + x_1}{2}, \frac{y + y_1}{2}\right) = \left(\frac{x}{2} + \frac{x_1}{2}, \frac{y}{2} + \frac{y_1}{2}\right) =$$

$$= \left(\frac{x}{2}, \frac{y}{2}\right) + \left(\frac{x_1}{2}, \frac{y_1}{2}\right) \Rightarrow (x,y) \to \begin{bmatrix} x \\ y \end{bmatrix}$$

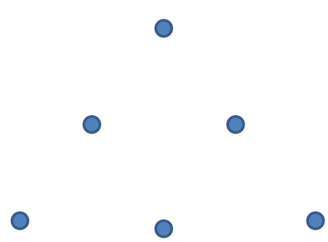
$$T_1(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x_1/2 \\ y_1/2 \end{bmatrix}$$

$$T_1(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, T_2(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/4 \\ 1/2 \end{bmatrix}, T_3(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$$

Qual a lógica?

$$T_1(0,0) = (0,0) \qquad T_1(1/2,1) = (1/4,1/2) \qquad T_1(1,0) = (1/2,0)$$
$$T_2(0,0) = (1/4,1/2) \qquad T_2(1/2,1) = (1/2,1) \qquad T_2(1,0) = (3/4,1/2)$$
$$T_3(0,0) = (1/2,0) \qquad T_3(1/2,1) = (3/4,1/2) \qquad T_3(1,0) = (1,0)$$

$$P_2 = \left(\frac{1}{2}, 1\right)$$



$P_1 = (0,0)$ $\qquad$ $P_3 = (1,0)$

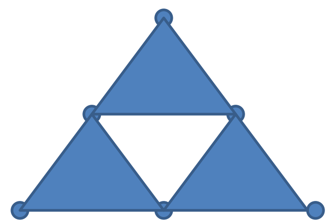
Qual a lógica?

$T_1(0,0) = (0,0)$ $\quad$ $T_1(1/2,1) = (1/4,1/2)$ $\quad$ $T_1(1,0) = (1/2,0)$

$T_2(0,0) = (1/4,1/2)$ $\quad$ $T_2(1/2,1) = (1/2,1)$ $\quad$ $T_2(1,0) = (3/4,1/2)$

$T_3(0,0) = (1/2,0)$ $\quad$ $T_3(1/2,1) = (3/4,1/2)$ $\quad$ $T_3(1,0) = (1,0)$

$$P_2 = \left(\frac{1}{2}, 1\right)$$

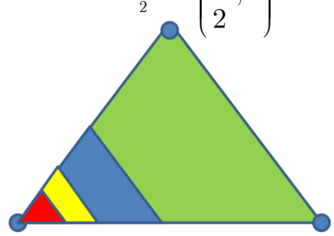


$P_1 = (0,0)$ $\quad$ $P_3 = (1,0)$

Qual a lógica?

$T_1(0,0) = (0,0)$ $\quad$ $T_1(1/2,1) = (1/4,1/2)$ $\quad$ $T_1(1,0) = (1/2,0)$

$$P_2 = \left(\frac{1}{2}, 1\right)$$

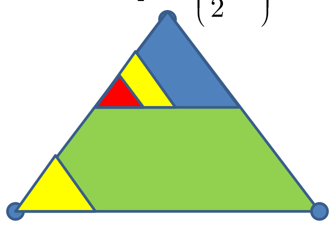


$$T_1(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$P_1 = (0,0)$ $\quad$ $P_3 = (1,0)$

Qual a lógica?

$$P_2 = \left(\frac{1}{2}, 1\right)$$



$$P_1 = \left(0, 0\right) \qquad P_3 = \left(1, 0\right)$$

$$T_1\left(x, y\right) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$T_2\left(x, y\right) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/4 \\ 1/2 \end{bmatrix}$$

Detalhes matemáticos

$$T\left(x, y\right) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$T^2\left(x, y\right) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \left(\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \right) + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} =$$

$$= \begin{bmatrix} 1/4 & 0 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} =$$

$$= \begin{bmatrix} 1/4 & 0 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + T\left(x_0, y_0\right)$$

Detalhes matemáticos

$$T^{2}(x,y)=\begin{bmatrix}1/4 & 0\\ 0 & 1/4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+T(x_0,y_0)$$

$$T^{3}(x,y)=\begin{bmatrix}1/2 & 0\\ 0 & 1/2\end{bmatrix}\left(\begin{bmatrix}1/4 & 0\\ 0 & 1/4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+T(x_0,y_0)\right)+\begin{bmatrix}x_0\\ y_0\end{bmatrix}$$

$$=\begin{bmatrix}1/8 & 0\\ 0 & 1/8\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+T^{2}(x_0,y_0)$$

$$T^{n}(x,y)=\begin{bmatrix}1/2^{n} & 0\\ 0 & 1/2^{n}\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+T^{n-1}(x_0,y_0)$$

Detalhes matemáticos

$$T^{n}(x,y)=\begin{bmatrix}1/2^{n} & 0\\ 0 & 1/2^{n}\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+T^{n-1}(x_0,y_0)$$

$$\text{Quando } n\rightarrow\infty:\quad T^{n}(x,y)\approx T^{n-1}(x_0,y_0)$$

$$T^{n}(x_0,y_0)=\sum_{k=1}^{k=n}\frac{1}{2^{k}}\begin{bmatrix}x_0\\ y_0\end{bmatrix}+\begin{bmatrix}x_0\\ y_0\end{bmatrix}=\left\{1+\sum_{k=1}^{k=n}\left(\frac{1}{2}\right)^{k}\right\}\begin{bmatrix}x_0\\ y_0\end{bmatrix}$$

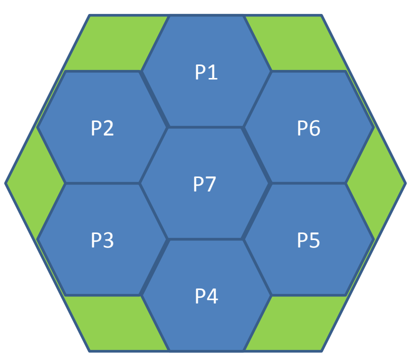
$$\text{Quando } n\rightarrow\infty:$$

$$T^{*}(x_0,y_0)=\left\{1+\frac{1/2}{1-1/2}\right\}\begin{bmatrix}x_0\\ y_0\end{bmatrix}=2\begin{bmatrix}x_0\\ y_0\end{bmatrix}$$

Detalhes matemáticos

$$T^*(x_0, y_0) = \left\{1 + \frac{q}{1-q}\right\}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \frac{1}{1-q}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

Outra figura q=1/3



$$d_f = \frac{\log 7}{\log 3} \approx 1,77$$

$$P_1 = (0, 0) \qquad P_4 = \left(0, \frac{1}{3}\right)$$

$$P_2 = \left(-\frac{\sqrt{3}}{12}, \frac{1}{9}\right) \qquad P_6 = \left(\frac{\sqrt{3}}{12}, \frac{1}{9}\right)$$

$$P_3 = \left(-\frac{\sqrt{3}}{12}, \frac{2}{9}\right) \qquad P_5 = \left(\frac{\sqrt{3}}{12}, \frac{2}{9}\right)$$

$$P_7 = \left(0, \frac{1}{6}\right) \qquad \sqrt{3}\,/\,12 \approx 0.144$$