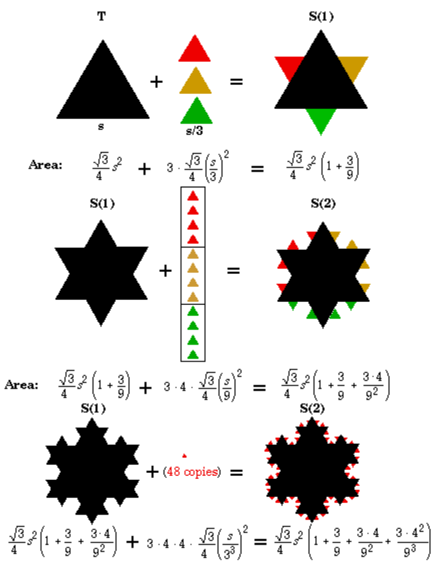
Atividade 3



Iteração	Lados	Termo
0	3	1
1	4 x 3	$3 \times 4\,\frac{1}{9^2}$
2	4 x 4 x 3	$3 \times 4^2\,\frac{1}{9^3}$
...		
k-1	$4^{k-1} \times 3$	$3 \times \frac{4^{k-2}}{9^{k-1}}$
k	$4^k \times 3$	$3 \times \frac{4^{k-1}}{9^k}$
...		

Atividade 3

$$A = 1 + \sum_{k=1}^{k=\infty} 3 \times \frac{4^{k-1}}{9^k} =$$

$$= 1 + \frac{3}{4} \sum_{k=1}^{k=\infty} \left(\frac{4}{9}\right)^k =$$

$$= 1 + \frac{3}{4} \times \frac{4}{9} \times \frac{1}{1 - \frac{4}{9}} =$$

$$= 1 + \frac{3}{9} \times \frac{9}{5} = \frac{8}{5}$$

Lembre: as séries geométricas com razão menor que 1 são o exemplo que vocês conhecem de "soma infinita" que dá resultado finito. Vocês vão ter muitos outros exemplos nos cursos em Nível Superior.

Iteração	Lados	Termo
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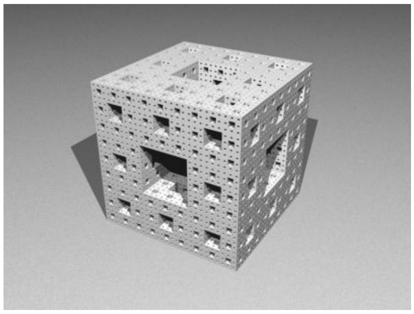
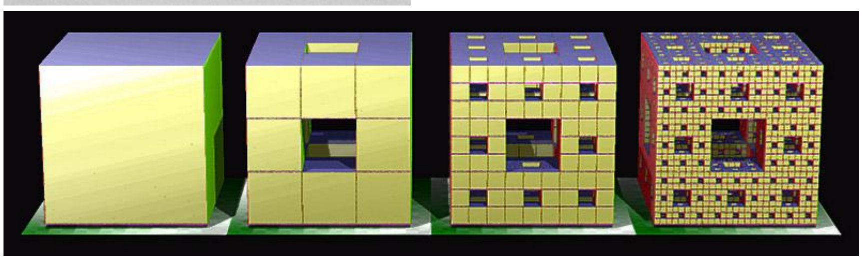
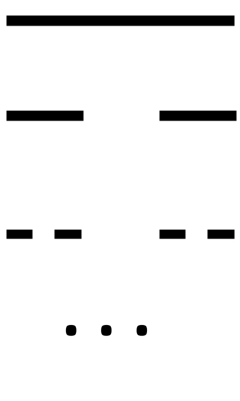
...		

Atividade 4



$$d_f = \frac{\log N}{\log\left(\frac{1}{r}\right)} = \frac{\log 20}{\log\left(1/\frac{1}{3}\right)} = \frac{\log 20}{\log(3)} \approx 2.72$$





$$C_1 = [0,1]$$

$$C_2 = [0,\frac{1}{3}] \cup [\frac{2}{3},1]$$

$$C_3 = \left([0,\frac{1}{9}] \cup [\frac{2}{9},\frac{1}{3}]\right) \cup \left([\frac{2}{3},\frac{7}{9}] \cup [\frac{8}{9},1]\right)$$

...

$$C_n = \left([0,\frac{1}{3^{n-1}}] \cup [\frac{2}{3^{n-1}},\frac{1}{3^{n-2}}]\right) \cup \ldots \cup \left([\frac{2}{3^{n-2}},\frac{3^{n-2}-2}{3^{n-1}}] \cup [\frac{3^{n-2}-1}{3^{n-1}},1]\right)$$

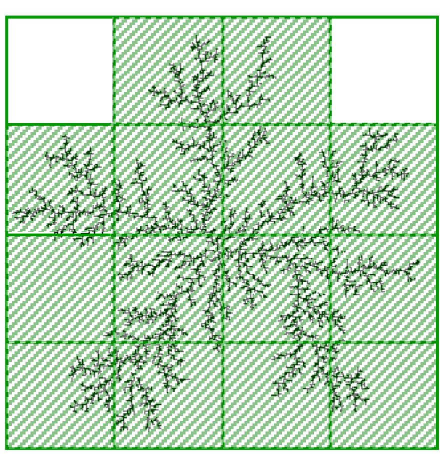
O **Conjunto de Cantor** é o que fica quando você “termina” o processo.

Em outras palavras:

$$C = \text{Conjuto de Cantor} = \bigcap_{n=1}^{\infty} C_n$$

Measuring fractal dimension

Box-counting: resolution-dependent measurement of D



< ∈ >

· cover the object by boxes of size ∈

· count non-empty boxes

· repeat for many ∈

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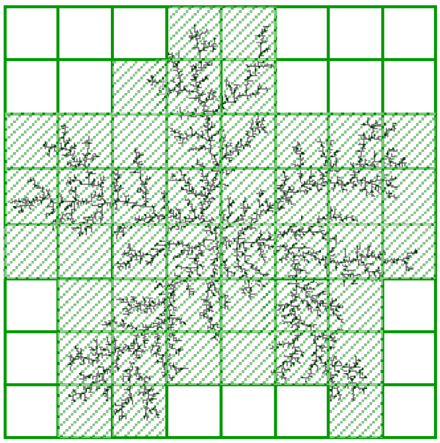
6

Measuring fractal dimension

box-counting: resolution-dependent measurement

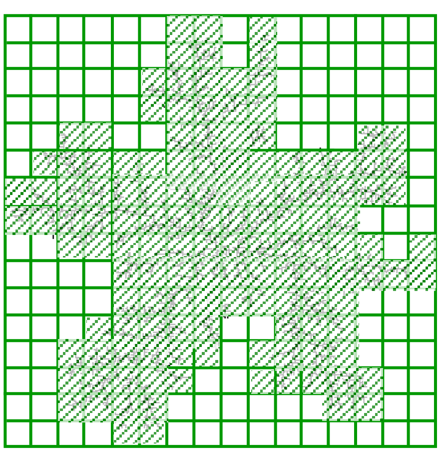


- cover the object by boxes of size ∊
- count non-empty boxes
- repeat for many ∊

7

Measuring fractal dimension

box-counting: resolution-dependent measurement

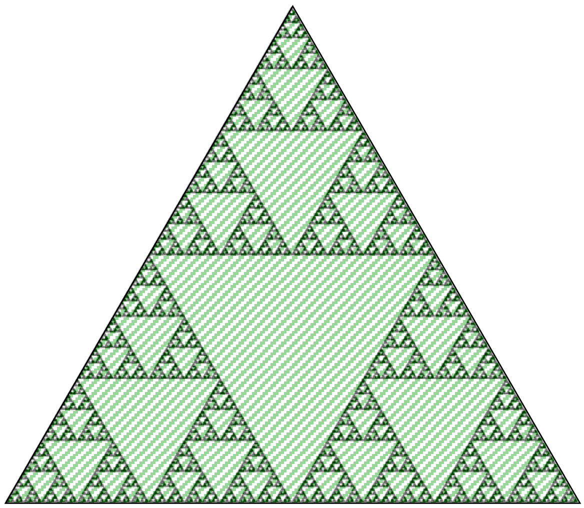


- cover the object by boxes of size ∊
- count non-empty boxes
- repeat for many ∊
- consider the number n of non-empty boxes as a function of ∊

(in the limit ∊→0)

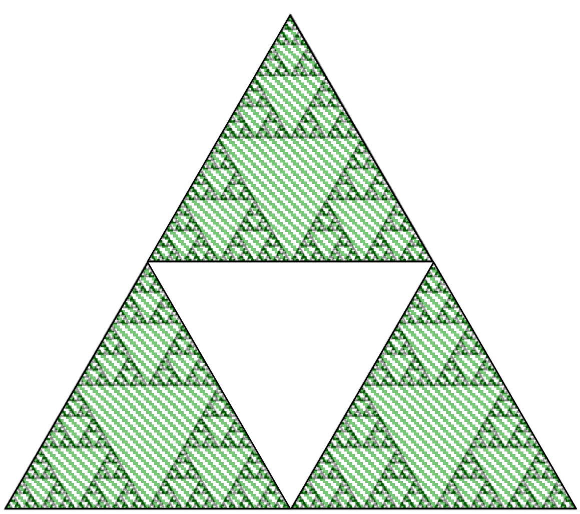
8

Sierpinsky revisited



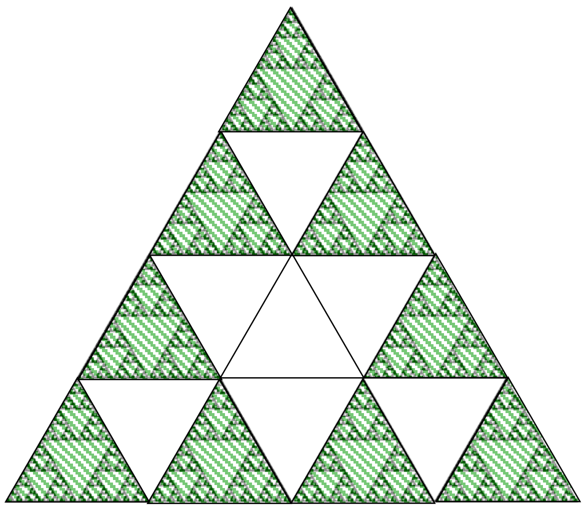
ϵ	n
1 1	

Sierpinsky revisited



ϵ	n
1 1	
1/2	3

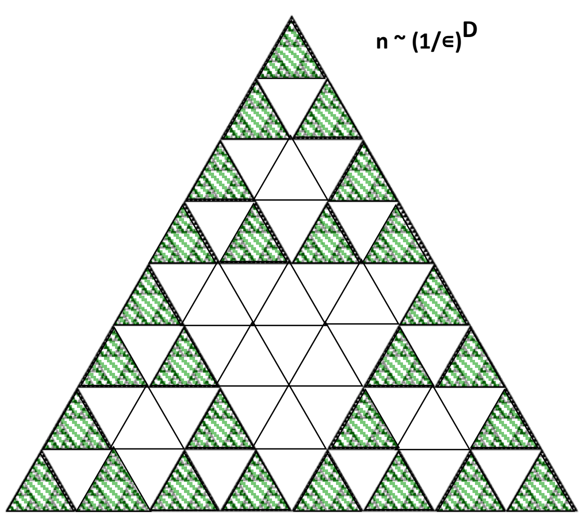
Sierpinsky revisited



∈	n
1 1	
1/2	3
1/4	9

11

Sierpinsky revisited



n ~ $(1/\in)^D$

k	∈	n
0	1 1	
1	1/2	3
2	1/4	9
3	1/8	27

$1/\in = 2^k$

$n = 3^k$

$$D = \frac{k \log(3)}{k \log(2)}$$

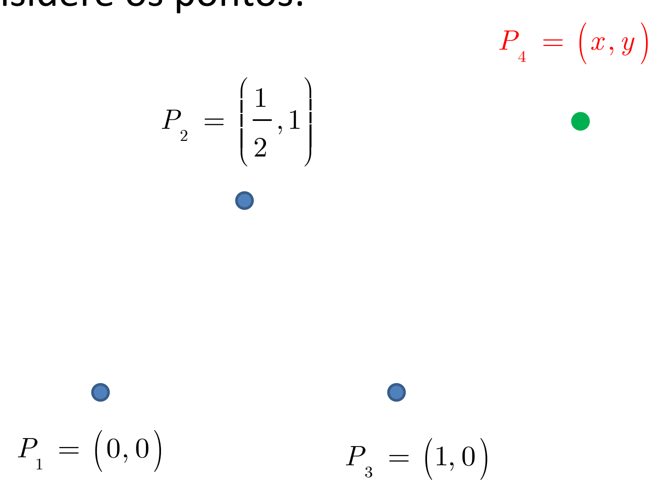
12

Transformações, matrizes e fractais

Deilson de Melo Tavares
EC&T/UFRN

Precisa-se de um palpite lotérico

Considere os pontos:



Qual a lógica da coisa?

$$T_1(x,y) = \left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right), \quad T_2(x,y) = \left(\frac{x+x_2}{2}, \frac{y+y_2}{2}\right), \quad T_3(x,y) = \left(\frac{x+x_3}{2}, \frac{y+y_3}{2}\right)$$

$$T_1(x,y) = \left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right) = \left(\frac{x}{2}+\frac{x_1}{2}, \frac{y}{2}+\frac{y_1}{2}\right) =$$

$$= \left(\frac{x}{2}, \frac{y}{2}\right) + \left(\frac{x_1}{2}, \frac{y_1}{2}\right) \Rightarrow (x,y) \to \begin{bmatrix} x \\ y \end{bmatrix}$$

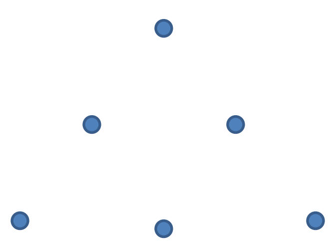
$$T_1(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x_1/2 \\ y_1/2 \end{bmatrix}$$

$$T_1(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, T_2(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/4 \\ 1/2 \end{bmatrix}, T_3(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$$

Qual a lógica?

$$T_1(0,0) = (0,0) \qquad T_1(1/2,1) = (1/4,1/2) \qquad T_1(1,0) = (1/2,0)$$
$$T_2(0,0) = (1/4,1/2) \qquad T_2(1/2,1) = (1/2,1) \qquad T_2(1,0) = (3/4,1/2)$$
$$T_3(0,0) = (1/2,0) \qquad T_3(1/2,1) = (3/4,1/2) \qquad T_3(1,0) = (1,0)$$

$$P_2 = \left(\frac{1}{2}, 1\right)$$



$$P_1 = (0,0) \qquad P_3 = (1,0)$$